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In some instances group comparisons in terms of upper or lower portions of the score distributions are more informative than comparisons of central tendency. These comparisons can be done by carrying out a split on the data prior to an analysis of variance (ANOVA). The resulting test statistic from ANOVA is not distributed as an F ratio however, and requires evaluation for significance relative to an empirical monte-carlo distribution. An example and computer program are presented.

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Evaluation of bootstrap and parametric percentile contrasts - Volume I

Splits Analysis: A Method for Noncentral Tendency Comparisons

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Running Head: SPLITS ANALYSIS

Abstract

In some instances group comparisons in terms of upper or lower portions of the score distributions are more informative than comparisons of central tendency. These comparisons can be done by carrying out a split on the data prior to an analysis of variance (ANOVA). The resulting test statistic from ANOVA is not distributed as an F ratio however, and requires evaluation for significance relative to an empirical monte-carlo distribution. An example and computer program are presented.

Splits Analysis: A Method for

Noncentral Tendency Comparisons

In the behavioral sciences, comparison of groups typically concerns contrast of central tendency. For example, a researcher interested in the effects of violent vs nonviolent tv programs would typically compare the mean, or perhaps the median, subsequent aggression of the first group vs the second group. There are instances, however when researchers would be interested not with central tendency differences; rather they would be concerned with differences in say the upper ten percent or the lower third of each group. For example, an industrial psychologist may want to investigate the efficacy of two training techniques in resultant skill acquisition. Since only the top ten percent of trainees may be hired or promoted, the two techniques would be best evaluated in terms of their effect on the upper ten percent of each group.

Lunneborg (1986) has described a bootstrap quantile analysis appropriate for comparing two groups at given percentiles. His procedure yields a probability value that the two groups' scores at a given percentile differ by chance. The present work describes an alternative method to Lunneborg's bootstrap procedure, and provides a computer implementation of the procedure. The alternative is referred to as a splits analysis as it concerns carrying out a split on the data prior to statistical analysis.

Consider a data set for which we are interested in comparing the upper half of the score distributions of two groups. To carry out the splits analysis the data would be rank ordered within groups, a median split would be carried out on each group, and the upper half of the data would be analyzed using a one-way analysis of variance (ANOVA). Although an ANOVA is carried out on the data the resulting test statistic is not evaluated using standard _F_ tables. Research by the author has indicated that such an approach would lead to a great inflation in the Type I error rate. For example, using standard _F_ distribution critical values typically resulted in actual Type I error rates in excess of .20 for the nominal .05 significance level (Rasmussen, 1990).

Instead of using the _F_ distribution, monte carlo methods are used to evaluate the significance of the obtained test statistic. Specifically, a large number, say 5000, data sets of the same sample size would be generated under the null hypothesis using a pseudo-random normal deviate generator. Each of the data sets would be processed identically as the original data set, i.e., an empirical test score distribution under the null hypothesis for the same sample size and data split would be created. The obtained test statistic is then evaluated relative to this empirical distribution to determine significance. For example, if 10 of the monte carlo values are larger than the obtained value then the probability value associated with the obtained value would be 10/5000 = 0.002.

The previous example would be roughly analogous to Lunneborg's bootstrap comparison of the 75th percentiles. Initial research by the author has indicated that the splits analysis approach maintained the .01 and .05 alpha levels, whereas the bootstrap procedure tended to be overly conservative (Rasmussen, 1990).

Table 1 presents a small data set along with the results from a splits analysis. In the example, there are 9 cases per group and the splits analysis compares the lower third of each group. The ANOVA test

Insert Table 1 about Here

statistic resulting from the splits analysis is 36.75. If this were an standard F ratio (i.e, with 1 and 4 degrees of freedom) it would have a probability value of .0037. The splits analysis probability of 0.016 is less extreme.

Similar to bootstrapping and approximate randomization procedures the probability value associated with splits analysis is an approximation that depends upon the number of monte carlo simulations and the significance level (Rasmussen, 1988; Rasmussen, 1989). With a known significance level, the formula for the standard error is $\underline{SE}$ = sqrt[$\underline{s}$ (1 - $\underline{s}$) / $\underline{m}$], where $\underline{s}$ is the significance level and $\underline{m}$ is the number of monte carlo simulations.

This formula can be used to evaluate the probability that a given approximate probability value is less than a desired probability value. For example the probability that the approximate probability value of 0.016 is less than a desired probability value of 0.05 can be calculated from $\underline{SE}$ = [.05 (1 - .05) / 5000] = .00308. Using the standard $\underline{z}$ score formula, $\underline{z}$ = (.016 - .05) / .00308 = -11.04. A $\underline{z}$ score of such magnitude indicates that it is extremely unlikely that the approximate probability value of 0.016 is greater than the 0.05 level. In instances in which the approximate probability value is close to the desired value, a larger number of simulations could be carried out.

<u>Program execution</u>

The program asks for the analysis parameters interactively and reads the data from a file. The program requires the sample size per group for the entire data set and the lower and upper ordinal values that represent the desired split. For example, for a sample size of 12 a lower value of 1 and an upper value of 3 would compare the lower quarter of the distributions, whereas a lower value of 9 and an upper value of 12 would compare the upper third of the distributions. The program also requests the number of monte carlo simulations to carry out. On the VAX the formula to estimate the execution time is Central Processing Unit (CPU) seconds = 7.4E-5 (<u>nm</u>), where n is the sample size per group after the data split. For example, with a sample size after the data split of 40 per group and with 20,000 repetitions it requires approximately 60 CPU seconds. Finally, the program requires the name of the data file. The data is read in groups using free format with one score per record. The program then carries out the appropriate split on the data and on the monte carlo simulations. The group means on the split data, the test statistic and the probability value is then calculated and printed out.

Table 2 gives the FORTRAN coding of the splits analysis along with an efficient ANOVA function. The program will require a random normal deviate generator

Insert Table 2 about Here

and an efficient sorting routine. These are readily available in Lehman (1977) and Miller (1982) or can be obtained from the author.

The program currently runs on a VAX 8800 computer. To run the program on another system it will probably be necessary to change the OPEN statement and the unit numbers associated with the READ and WRITE statements. In addition the SECNDS and RAN functions may be different on other systems. The SECNDS function is used to give a different series of random numbers based on the time in seconds since midnight. On systems which cannot readily provide a function to give the time the program can be modified to ask the user for a seed (e.g., a random nine digit odd number) to start the random number generator.

Author's note

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Table 1

Sample splits analysis

Group 1	Group 2		
13	28	Mean 1	Mean 2
16	29	16.0	30.0
19	33	Test Statistic:	36.75
....		Probability:	0.016
24	34		
29	38		
32	40		
36	41		
37	43		
42	45		

Table 2

Source Code for Splits Analysis

```
      REAL X(1000), Y(1000), FMC(100000)
      CHARACTER IFILE*20
      XXX = 1.0
      III = SECNDS(XXX) * 2000 + 1
      WRITE (6,19)
19    FORMAT('    This program calculates probability values'/
     1 ' for splits on data.  Give the sample size per group,'/
     1 ' upper and lower split values, and number of monte'/
     1 ' carlo trials. '/)
      READ (6,*) NPERG, ISPLTL, ISPLTU, NMC
      XNMC = NMC
      NSPLT = ISPLTU - ISPLTL + 1
      XNSPLT = NSPLT
      WRITE (6,29)
29    FORMAT('    Give the name of the data file '/)
      READ (6,39) IFILE
39    FORMAT(A20)
      OPEN (27, FILE = IFILE, STATUS = 'UNKNOWN')
      DO 10 I = 1, NPERG
      READ (27,*) X(I)
10    CONTINUE
      DO 20 I = 1, NPERG
      READ (27,*) Y(I)
20    CONTINUE
      CALL SORT (X, NPERG)
      CALL SORT (Y, NPERG)
      FOBS = ANOVA (X, Y, TOTX, TOTY, ISPLTL, ISPLTU, NSPLT)
      XMEAN = TOTX / XNSPLT
      YMEAN = TOTY / XNSPLT
      DO 40 IREP = 1, NMC
      DO 30 I = 1, NPERG
      X(I) = RNORM(III)
      Y(I) = RNORM(III)
30    CONTINUE
      CALL SORT (X, NPERG)
      CALL SORT (Y, NPERG)
      FMC(IREP) = ANOVA (X, Y, TOTX, TOTY, ISPLTL, ISPLTU, NSPLT)
40    CONTINUE
      CALL SORT (FMC, NMC)
      ITST = 0
```

Table 2 continues

Table 2, continued

Source Code for Splits Analysis

```
        DO 50 IREP = 1, NMC
        IF (FOBS .LT. FMC(IREP)) GOTO 51
        ITST = ITST + 1
50      CONTINUE
51      CONTINUE

        XTST = ITST
        PROB = (XNMC - XTST) / XNMC
        WRITE (6,49) XMEAN, YMEAN, FOBS, PROB
49      FORMAT(//'          Means:',2F12.4/
     1  ' Test Statistic:',F12.4/'    Probability:',F12.4//)
        STOP
        END
C
        FUNCTION ANOVA (X, Y, TOT1, TOT2, ISPLTL, ISPLTU, N)
        REAL X(1000), Y(1000)
        XN = N
        XNTOT = XN * 2.0
        DFW = 2.0 * (XN - 1.0)
        TOT1 = 0
        TOT2 = 0
        SXSQ = 0
        DO 10 I = ISPLTL, ISPLTU
        TOT1 = TOT1 + X(I)
        TOT2 = TOT2 + Y(I)
        SXSQ = SXSQ + X(I)**2 + Y(I)**2
10      CONTINUE
        TOTV = (TOT1**2 + TOT2**2) / XN
        CF = (TOT1 + TOT2)**2 / XNTOT
        SSB = TOTV - CF
        SSW = SXSQ - TOTV
        VMSW = SSW / DFW
        ANOVA = 0.0
        IF (VMSW .GT. 0.0) ANOVA = SSB / VMSW
        RETURN
        END
```
